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Soliton pulse propagation in optical Fiber

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1. The nonlinear schrodinger equation.

Pulse's transmission in optical fiber is described by equation

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \beta_1\frac{\partial A}{\partial t} - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} + i\gamma\left[|A|^2 A + \frac{i}{\omega_0}\frac{\partial}{\partial t}|A|^2 A - T_R A\frac{\partial\left(|A|^2\right)}{\partial t}\right] \quad (1)$$

Self- phase modulation
Nonlinear

Optical self-steepening
Nonlinear

Delayed Raman response
T_R=3-5 fs.
Nonlinear

For pulse width $T_0 > 5$ ps, one can use Eq (1) give by

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \beta_1 \frac{\partial A}{\partial t} - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + i\gamma |A|^2 A \qquad (2)$$

Applying the transformation $T = t - z/v_g$, (v_g is group velocity) equation (2) is re-written as follows:

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + i\gamma |A|^2 A \qquad (3)$$

Let us introduce a time scale normalized to the input pulse width T_0 as

$$\tau = \frac{T}{T_0} = \frac{1 - z/v_g}{T_0} \qquad (4)$$

At the same time, we introduce a normalized amplitude U as

$$A(z,\tau) = \sqrt{P_0}\exp\left(-\frac{\alpha z}{2}\right)U(z,\tau) \tag{5}$$

where P_0 is the peak power of the incident pulse.

The exponential factor in (5) accounts for fiber losses. By using Eqs. (4), (3), U is found to satisfy

$$i\frac{\partial U}{\partial z} = \frac{\operatorname{sgn}(\beta_2)}{2L_D}\frac{\partial^2 U}{\partial \tau^2} - \frac{\exp(-\alpha z)}{L_N}|U|^2 U, \tag{6}$$

where

the dispersion length $L_D = \dfrac{T_0^2}{|\beta_2|}$

the nonlinear length $L_{NL} = \dfrac{1}{\gamma P_0}$

Group-velocity dispersion (GVD)

The effect of GVD on optical pulses propagating in a linear dispersive medium are studied by setting $\gamma = 0$ in Eq (3). If we define the normalized amplitude U(z,T) according to Eq(5) U(z, T) satisfies the following linear partial differential equation:

$$i\frac{\partial U(z,T)}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 U(z,T)}{\partial T^2} \tag{7}$$

Equation (7) is readily solved by using the Fourier-transform method. If $U_1(z, \omega)$ is the Fourier transform of U(z; T) such that

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} U_1(z,\omega)\exp(-i\omega T)\,d\omega$$

then it satisfies an ordinary differential equation

$$i\frac{\partial U_1(z,\omega)}{\partial z} = -\frac{1}{2}\beta_2\omega^2 U(z,\omega)$$

whose solution is given by

$$U_1(z,\omega) = U_1(0,\omega)\exp\left(\frac{i}{2}\beta_2\omega^2 z\right)$$

and

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} U_1(0,\omega)\exp\left(\frac{i}{2}\beta_2\omega^2 t - i\omega T\right)d\omega \qquad (8)$$

where

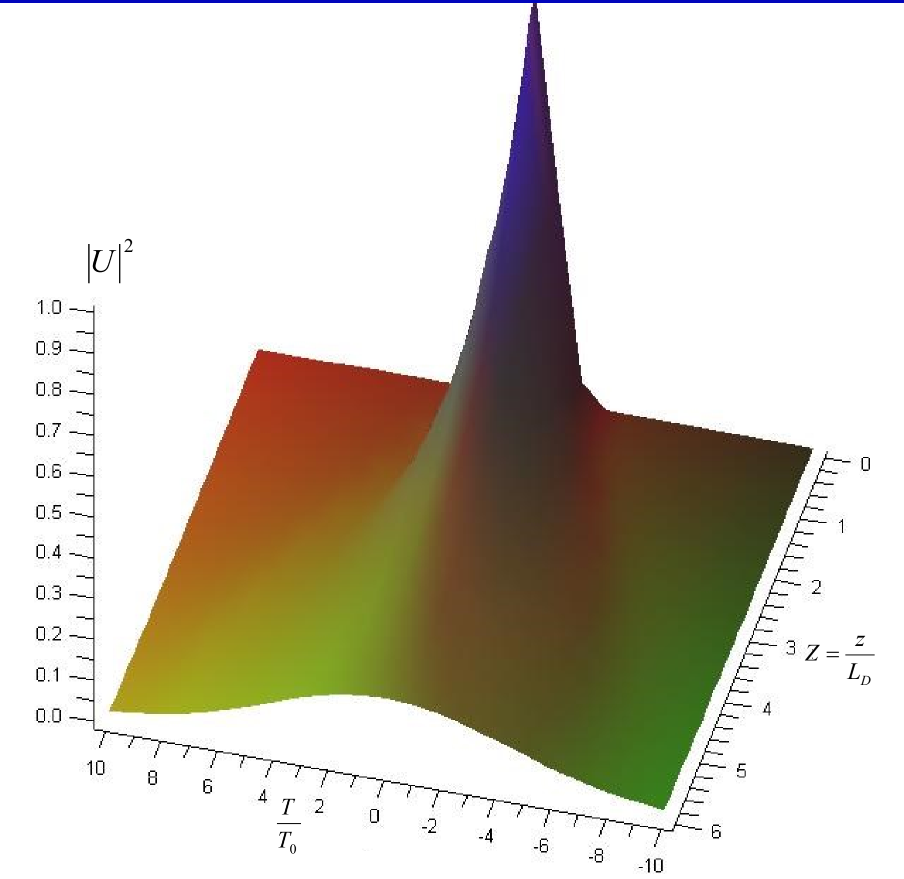
$$U_1(0,\omega) = \int_{-\infty}^{\infty} U(0,T)\exp(i\omega T)dT \qquad (9)$$

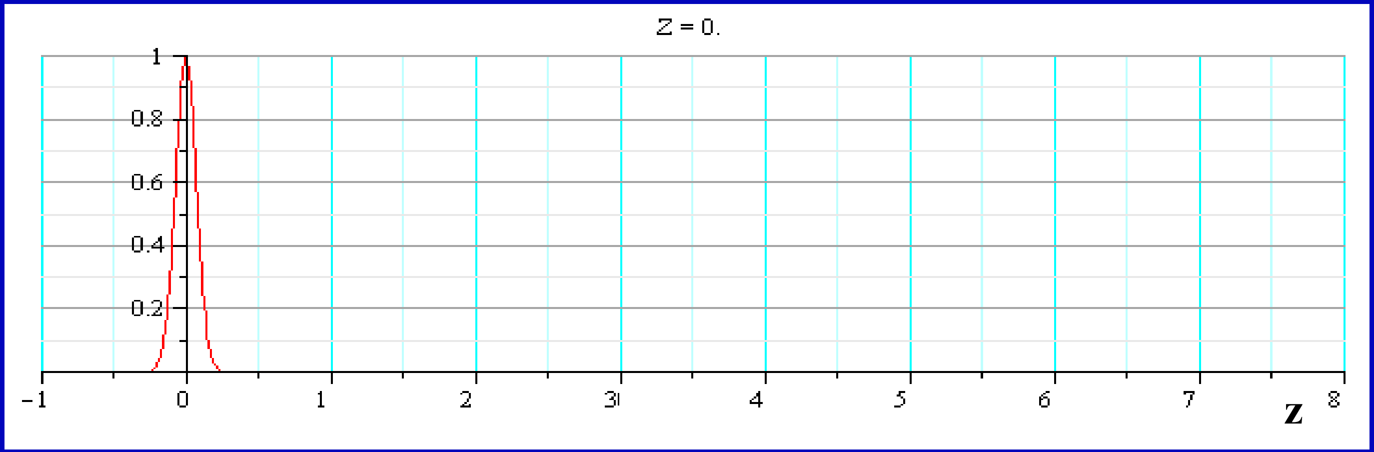
Equations (8) and (9) can be used for input pulses of arbitrary shapes.

example $$U(0,T)=\exp\left(-\frac{T^2}{2T_0^2}\right) \tag{10}$$

We have $$U(z,T)=\frac{T_0}{\left(T_0^2-i\beta_2 z\right)^{1/2}}\exp\left(-\frac{T^2}{2\left(T_0^2-i\beta_2 z\right)}\right) \tag{11}$$

Fig1. *3D graph of the transmission of pulse*





Self- phase modulation

In terms of the normalized amplitude U(z; T) defined as in Eq (5) the pulse-propagation equation (6), in the limit $\beta_2 = 0$, becomes

$$\frac{\partial U}{\partial z} = i \frac{\exp(-\alpha z)}{L_N} |U|^2 U, \tag{12}$$

Equation (12) can be solved substituting $U = V\exp(i\Phi_{NL})$ and equating the real and imaginary parts so that

$$\frac{\partial V}{\partial z} = 0; \qquad \frac{\partial \Phi_{NL}}{L_{LN}} = V^2. \tag{13}$$

As the amplitude V does not change along the fiber length L, the phase equation can be integrated analytically to obtain the general solution

$$U(L,T) = U(0,T)\exp\left[i\Phi_{NL}(L,T)\right], \tag{14}$$

where

$$\Phi_{NL}(L,T) = \left|U(0,T)\right|^2 \left(\frac{L_{eff}}{L_{LN}}\right), \tag{15}$$

with the effective length L_{eff} defined as

$$L_{eff} = \left[\frac{1 - \exp(-\alpha L)}{\alpha}\right] \tag{16}$$

Equation (14) shows that SPM gives rise to an intensity-dependent phase shift but the pulse shape remains unaffected.

In the absence of fiber losses, $\alpha=0$, and $L_{eff}= L$. The maximum phase shift Φ_{max} occurs at the pulse center located at T=0. With U normalized such $|U(0,0)|=1$, it is given by

$$\Phi_{max} = \frac{L_{eff}}{L_{NL}} = \gamma P_0 L_{eff} \tag{17}$$

The physical meaning of the nonlinear length L_{NL} is clear from Eq. (17)—it is the effective propagation distance at which $\Phi_{max}= 1$

The SPM-induced spectral broadening is a consequence of the time dependence of Φ_{NL}. This can be understood by noting that a temporally varying phase implies that the instantaneous optical frequency differs across the pulse from its central value ω_0. The difference $\delta\omega$ is given by

$$\delta\omega(T) = -\frac{\partial \Phi_{NL}}{\partial T} = -\left(\frac{L_{eff}}{L_{NL}}\right)\frac{\partial}{\partial T}\left|U(0,T)\right|^2$$

example $U(0,T) = \exp\left(-\frac{T^2}{2T_0^2}\right)$

We have

$$\delta\omega(T) = \frac{T}{T_0^2}\left(\frac{L_{eff}}{L_{NL}}\right)\exp\left(-\frac{T^2}{T_0^2}\right)$$

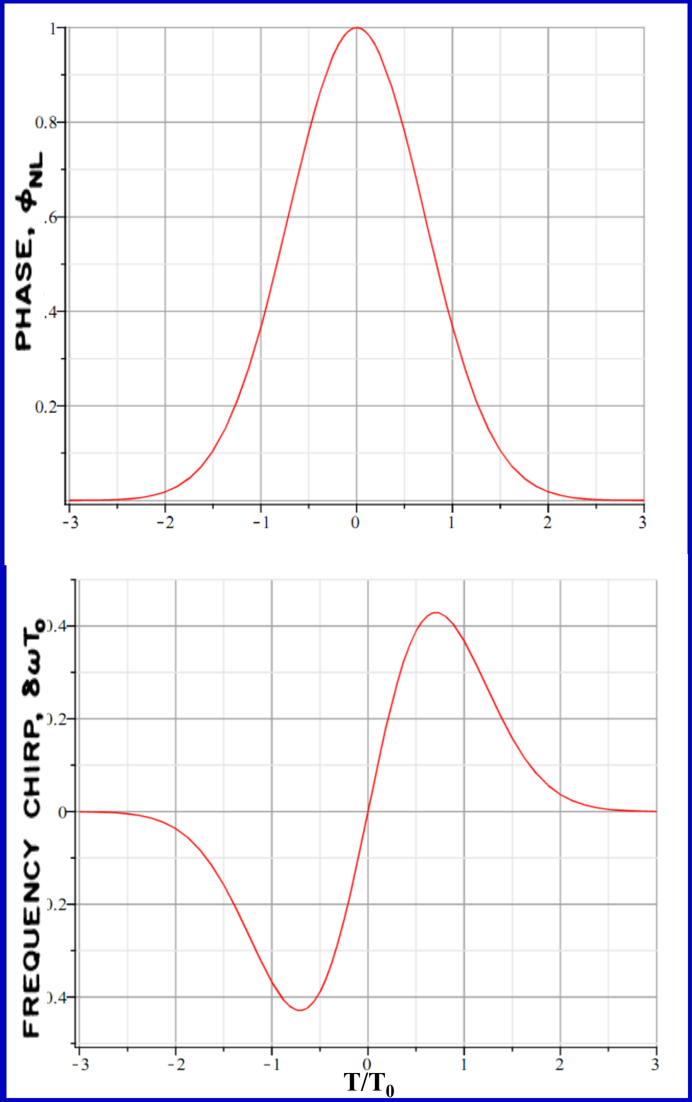


Figure 2 Temporal variation of SPM-induced phase shift Φ_{NL} and frequency chirp $\delta\omega$ for Gaussian

Split-Step Fourier Method

To understand the philosophy behind the split-step Fourier method, it is usefulto write Eq. (1) formally in the form

$$i\frac{\partial A(z,t)}{\partial z} = \hat{D}A(z,t) + \hat{N}(A).A(z,t) \tag{19}$$

Where

$$\hat{D} = -\frac{\alpha}{2} - \beta_1\frac{\partial}{\partial t} - \frac{i\beta_2}{2}\frac{\partial^2}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3}{\partial t^3} \tag{20}$$

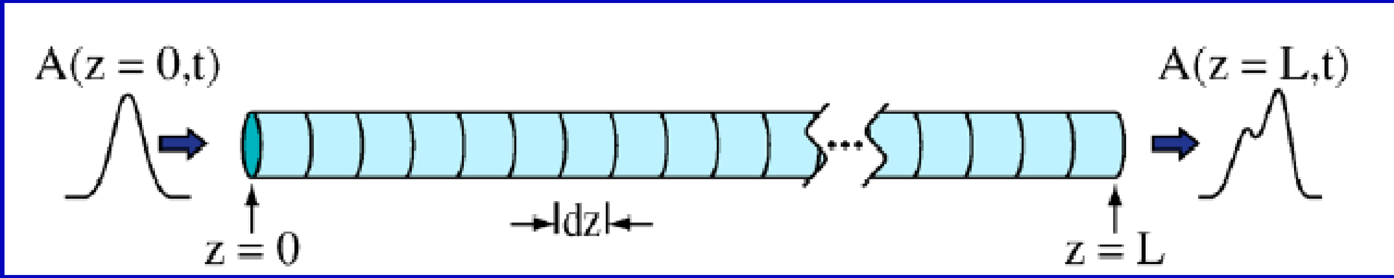
is an operator containing time derivative

$$\hat{N}(A) = i\gamma\left[|A|^2 + \frac{i}{\omega_0}\frac{\partial}{\partial t}|A|^2 - T_R\frac{\partial\left(|A|^2\right)}{\partial t}\right] \tag{21}$$

is a non-linear operator and is a function of A(z,t).

Solution of equation (19) has the form

$$A(z+dz,t)=\exp -i\left[\hat{L}dz+\hat{N}(A)dz\right]A(z,t) \tag{22}$$



$$A(z+dz)=\exp\left(\frac{dz}{2}.\hat{D}\right)\exp\left[dz.\hat{N}\left(\exp\left(\frac{dz}{2}.\hat{D}\right)A(z)\right)\right]\exp\left(\frac{dz}{2}.\hat{L}\right)A(z) \tag{23}$$

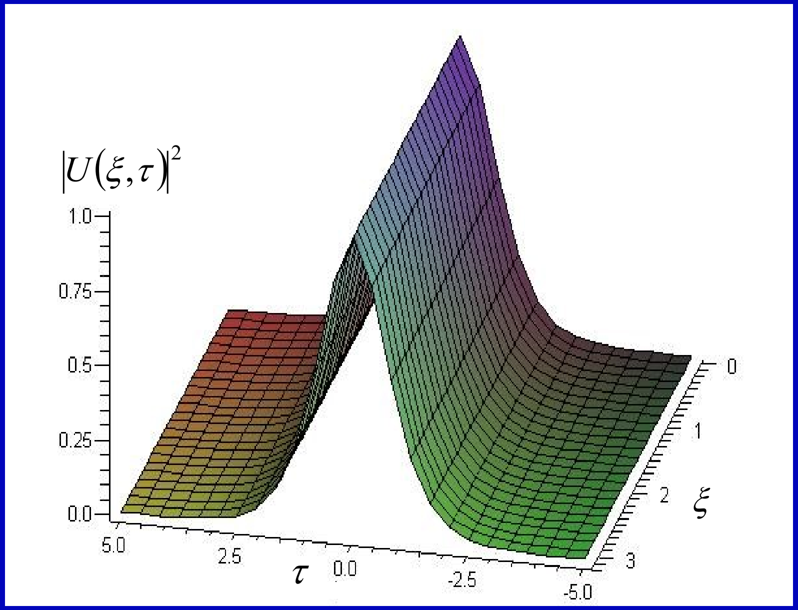
3. Results

So that we can normalize the equation (6) to obtain

$$i\frac{\partial U}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} - |U|^2 U, \tag{24}$$

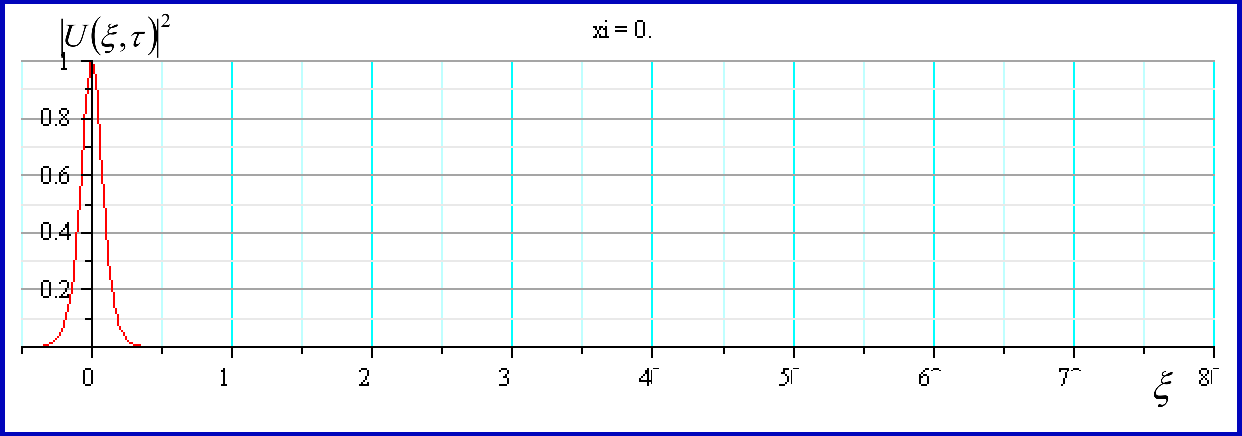
$$\xi = \frac{z}{L_D}, \beta_2 < 0$$

Using Split-Step Fourrier method, we obtain the following results:



The time-shape of intensity and the changing of this soliton on propagation path are described in fig 3. From this fig, can see that the first-order soliton has the time-shape of sech function and it is not verying along the propagation path. As we hope this optical soliton can be used for optical communication.

Fig 3. Revolution of first-order soliton in time-space



second – order soliton:

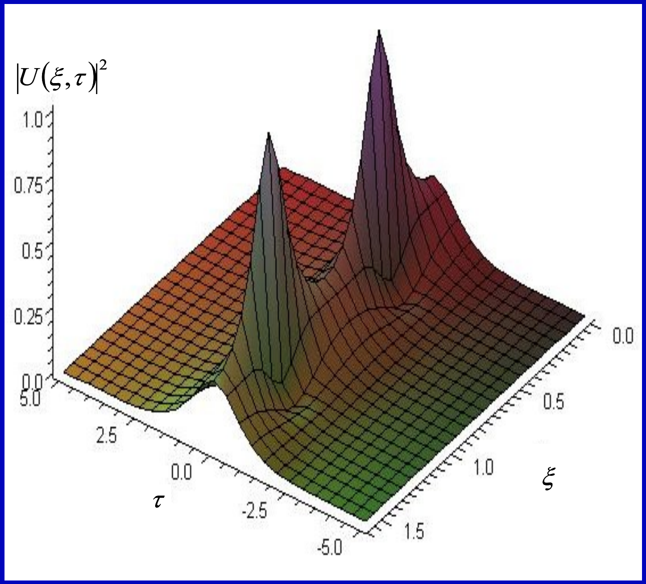


Fig 4. Revolution of second-order soliton in time-space

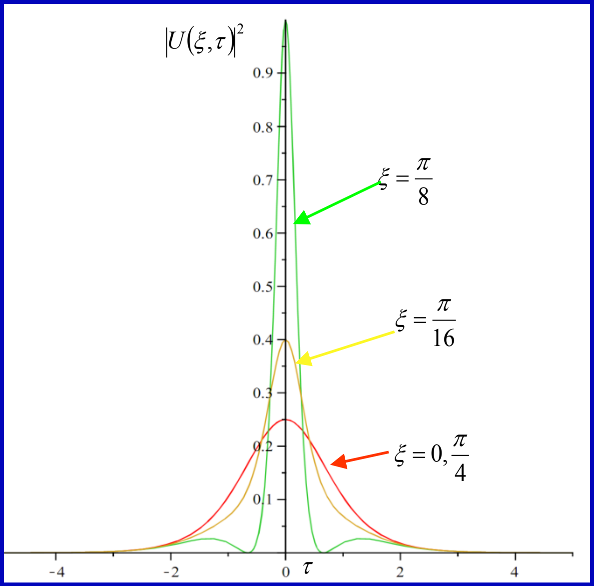


Fig.5. Time-shape of intensity of second-order soliton with some value of ξ

The time – shape of intensity is described in fig 5. From this fig can see that the the time-shape of intensity is verying a little to the first-order soliton. But the verying of it on the propagation path (see fig 4) is almost different. It has a intensity- verying period (π/4).